

# Lifetimes and Sizes from Two-Particle Correlation Functions

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## Abstract

We discuss the Yano-Koonin-Podgoretskii (YKP) parametrization of the two-particle correlation function for azimuthally symmetric expanding sources. We derive model-independent expressions for the YKP fit parameters and discuss their physical interpretation. We use them to evaluate the YKP fit parameters and their momentum dependence for a simple model for the emission function and propose new strategies for extracting the source lifetime. Longitudinal expansion of the source can be seen directly in the rapidity dependence of the Yano-Koonin velocity.

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The two-particle correlation functions  $C(\mathbf{p}_1, \mathbf{p}_2)$  of identical particles provide direct access to the spatio-temporal evolution of the collision region in heavy-ion collisions. This follows from the basic relation [2–5] between the correlation function  $C$  and the emission function  $S(x, K)$  (here written down for bosons)

$$C(\mathbf{K}, \mathbf{q}) \approx 1 + \frac{|\int d^4x S(x, K) e^{iq \cdot x}|^2}{|\int d^4x S(x, K)|^2}. \quad (1)$$

The emission function  $S$  describes the phase space (Wigner) density of the boson emitting sources, and  $q = p_1 - p_2$ ,  $K = (p_1 + p_2)/2$  (with  $p_1, p_2$  being on-shell such that  $K \cdot q = 0$ ) correspond to the relative and average 4-momenta of the boson pair. Eq. (1) neglects final state interactions; for a recent review of methods to include the latter see Ref. [6]. All other approximations leading to (1) are well-controlled [4,7].

It is the aim of Hanbury-Brown/Twiss (HBT) interferometry to extract information about the space-time characteristics of  $S(x, K)$  from the measured two-particle momentum correlations by “inverting” Eq. (1). Unfortunately, due to the on-shell conditions for the individual particle momenta  $p_1, p_2$ , this is not possible in a completely model-independent way: the time component  $q^0$  of the relative momentum is constrained by

$$q^0 = \boldsymbol{\beta} \cdot \mathbf{q}, \quad \boldsymbol{\beta} = \frac{\mathbf{K}}{K^0} = \frac{2\mathbf{K}}{(E_1 + E_2)} \approx \frac{\mathbf{K}}{E_K} \quad (2)$$

(with  $E_K = \sqrt{m^2 + \mathbf{K}^2}$  in the last step [7]), and thus the inverse Fourier transform of Eq. (1) is not unique.

In practice the analysis of HBT correlation data must therefore be based on a comparison with specific models for the source function  $S(x, K)$ , with the aim of eliminating “unreasonable” models and fitting certain essential parameters (geometric size, freeze-out temperature, collective flow velocity, time duration of the particle emission process) in a class of “reasonable” model sources. This procedure is enormously simplified by using so-called “model-independent” expressions for the HBT parameters [7–9] which allow to calculate from an arbitrary source function  $S$  the characteristic parameters of the two-particle correlation function  $C$  by simple quadrature.

While it is obvious from its definition that the single-particle spectrum is nothing but the zeroth space-time moment of the emission function,

$$E_K \frac{dN}{d^3K} = \int d^4x S(x, K), \quad (3)$$

it can also be shown [7,9,11] that the two-particle correlation function is essentially determined by its (normalized) second order space-time moments. Specifically, to compute the correlation function  $C$  it is sufficient to approximate the source function  $S$  by a Gaussian which contains only information on its space-time moments up to second order. To see this we write

$$S(x, K) = N(\mathbf{K}) S(\bar{x}(\mathbf{K}), K) \exp \left[ -\frac{1}{2} \tilde{x}^\mu(\mathbf{K}) B_{\mu\nu}(\mathbf{K}) \tilde{x}^\nu(\mathbf{K}) \right] + \delta S(x, K), \quad (4)$$

where

$$\bar{x}^\mu(\mathbf{K}) = \langle x^\mu \rangle, \quad \tilde{x}^\mu(\mathbf{K}) = x^\mu - \bar{x}^\mu(\mathbf{K}), \quad (B^{-1})_{\mu\nu}(\mathbf{K}) = \langle \tilde{x}_\mu \tilde{x}_\nu \rangle, \quad (5)$$

with the ( $\mathbf{K}$ -dependent) expectation values defined as space-time averages over the source function

$$\langle f(x) \rangle = \frac{\int d^4x f(x) S(x, K)}{\int d^4x S(x, K)}. \quad (6)$$

Then the term  $\delta S$  has vanishing zeroth, first and second order moments and thus contains only higher order information on sharp edges, wiggles, secondary peaks, etc. in the source. It was shown numerically [11] to have negligible influence on the half width of the correlation function and to contribute only weak, essentially unmeasurable structures in  $C(\mathbf{K}, \mathbf{q})$  at large values of  $\mathbf{q}$ . Neglecting  $\delta S$ , the single-particle spectrum (3) and the two-particle correlation function (1) can be calculated analytically:

$$E_K \frac{dN}{d^3K} = N(\mathbf{K}) \det \left( (B^{-1})_{\mu\nu}(\mathbf{K}) \right) S(\bar{x}(\mathbf{K}), K), \quad (7)$$

$$C(\mathbf{K}, \mathbf{q}) = 1 + \exp \left[ -q^\mu q^\nu \langle \tilde{x}_\mu \tilde{x}_\nu \rangle(\mathbf{K}) \right]. \quad (8)$$

The factor  $\det(B^{-1}(K))$  in (7) can be interpreted [10] as the generalized 4-volume of the emission region for particles with momentum  $K$ ,  $V_*^{(4)}(\mathbf{K}) = \det(\langle \tilde{x}_\mu \tilde{x}_\nu \rangle)$ . However, due

to the  $K$ -dependent normalization factor  $N(K)$ , neither  $V_*^{(4)}(\mathbf{K})$  nor the point  $\bar{x}^\mu(\mathbf{K})$  of maximum emissivity at momentum  $\mathbf{K}$  can be uniquely unfolded from the single-particle spectrum; the latter also drops out from the two-particle correlation function. Only the ( $\mathbf{K}$ -dependent) effective widths (“lengths of homogeneity” [12,7])  $\langle \tilde{x}_\mu \tilde{x}_\nu \rangle(\mathbf{K})$  of the source of particles with momentum  $\mathbf{K}$  are accessible by HBT interferometry.

Furthermore, due to the on-shell constraint (2), only 6 linear combinations of the variances  $\langle \tilde{x}_\mu \tilde{x}_\nu \rangle(\mathbf{K})$  are actually measurable [13]; in the case of azimuthal symmetry of the source around the beam axis, this number reduces to 4. To make contact with experimental correlation data, the redundant components must be eliminated from the exponent of the Gaussian in (8). It is convenient to do this by using a cartesian coordinate system with  $z$  along the beam axis and  $\mathbf{K}$  lying in the  $x$ - $z$ -plane. Customarily one labels the  $z$ -component of a 3-vector by  $l$  (for *longitudinal*), the  $x$ -component by  $o$  (for *outward*) and the  $y$ -component by  $s$  (for *sideward*). Then from (2) we see that  $\beta_s = 0$  such that

$$q^0 = \beta_\perp q_o + \beta_l q_l \quad (9)$$

with  $\beta_\perp = |\mathbf{K}_\perp|/K^0$  being (approximately) the velocity of the particle pair transverse to the beam direction while  $\beta_l$  is its longitudinal component.

The standard form [7,9] for the parametrization of the correlation function is obtained by using (9) to eliminate  $q^0$  from Eq. (8). One obtains

$$C(\mathbf{K}, \mathbf{q}) = 1 + \exp \left[ - \sum_{i,j=s,o,l} R_{ij}^2(\mathbf{K}) q_i q_j \right] \quad (10)$$

where the 6 HBT radius parameters  $R_{ij}$  are defined in terms of the following variances of the source function [9,7]:

$$R_{ij}^2(\mathbf{K}) = \langle (\tilde{x}_i - \beta_i \tilde{t})(\tilde{x}_j - \beta_j \tilde{t}) \rangle, \quad i, j = s, o, l. \quad (11)$$

For an azimuthally symmetric collision region or an azimuthally symmetric sample of collision events,  $C(\mathbf{K}, \mathbf{q})$  is symmetric with respect to  $q_s \rightarrow -q_s$  [13]. Then  $R_{os}^2 = R_{sl}^2 = 0$  and

$$C(\mathbf{K}, \mathbf{q}) = 1 + \exp \left[ -R_s^2(\mathbf{K})q_s^2 - R_o^2(\mathbf{K})q_o^2 - R_t^2(\mathbf{K})q_t^2 - 2R_{ot}^2(\mathbf{K})q_oq_t \right], \quad (12)$$

with

$$R_s^2(\mathbf{K}) = \langle \tilde{y}^2 \rangle, \quad (13a)$$

$$R_o^2(\mathbf{K}) = \langle (\tilde{x} - \beta_\perp \tilde{t})^2 \rangle, \quad (13b)$$

$$R_t^2(\mathbf{K}) = \langle (\tilde{z} - \beta_t \tilde{t})^2 \rangle, \quad (13c)$$

$$R_{ot}^2(\mathbf{K}) = \langle (\tilde{x} - \beta_\perp \tilde{t})(\tilde{z} - \beta_t \tilde{t}) \rangle. \quad (13d)$$

Clearly these HBT radius parameters mix spatial and temporal information on the source in a non-trivial way. Their interpretation in various reference systems, in particular the meaning of the generally non-vanishing cross-term  $R_{ot}^2$ , was extensively discussed in Refs. [7,9,11,13], by analyzing these expressions analytically for a large class of (azimuthally symmetric) model source functions and comparing with the numerically calculated correlation function (1). An important observation resulting from these studies is that the difference

$$R_{\text{diff}}^2 \equiv R_o^2 - R_s^2 = \beta_\perp^2 \langle \tilde{t}^2 \rangle - 2\beta_\perp \langle \tilde{x}\tilde{t} \rangle + (\langle \tilde{x}^2 \rangle - \langle \tilde{y}^2 \rangle) \quad (14)$$

is generally dominated by the first term on the r.h.s. and thus provides access to the lifetime  $\Delta t = \sqrt{\langle \tilde{t}^2 \rangle - \langle \tilde{t} \rangle^2}$  of the source [14] (more exactly: the duration of the particle emission process). However, in heavy-ion collisions, due to rapid expansion of the source one would not expect  $\langle \tilde{t}^2 \rangle$  to be generically much larger than either  $\langle \tilde{x}^2 \rangle$  or  $\langle \tilde{y}^2 \rangle$ ; in the situations investigated so far (e.g. [11]) it comes out an order of magnitude smaller. In the standard fit one is not sensitive to small values of  $\Delta t$  since Eq. (14) then involves a small difference of two large numbers, each associated with standard experimental errors. The factor  $\beta_\perp^2 \leq 1$  in front of  $\langle \tilde{t}^2 \rangle$  further complicates its extraction, in particular at low  $K_\perp$  where  $\Delta t(\mathbf{K})$  is usually largest (see below). Indeed, published experimental results [15,16] so far show no positive evidence for a finite duration of the particle emission process, in contradiction to all physical intuition.

We will show here that a generalization to azimuthally symmetric systems [13,17] of the Yano-Koonin parametrization for a moving source [18] circumvents this problem [19]. This

Yano-Koonin-Podgoretskii (YKP) form is based on an elimination in Eq. (8) of  $q_o$  and  $q_s$  in terms of  $q_\perp = \sqrt{q_o^2 + q_s^2}$ ,  $q^0$ , and  $q_3$ , using Eq. (9):

$$C(\mathbf{K}, \mathbf{q}) = 1 + \exp \left[ -R_\perp^2(\mathbf{K}) q_\perp^2 - R_\parallel^2(\mathbf{K}) (q_t^2 - (q^0)^2) - (R_0^2(\mathbf{K}) + R_\parallel^2(\mathbf{K})) (q \cdot U(\mathbf{K}))^2 \right], \quad (15)$$

where  $U(\mathbf{K})$  is a ( $K$ -dependent) 4-velocity with only a longitudinal spatial component:

$$U(\mathbf{K}) = \gamma(\mathbf{K}) (1, 0, 0, v(\mathbf{K})), \quad \text{with } \gamma = \frac{1}{\sqrt{1 - v^2}}. \quad (16)$$

This parametrization has the advantage that the YKP parameters  $R_\perp^2(\mathbf{K})$ ,  $R_0^2(\mathbf{K})$ , and  $R_\parallel^2(\mathbf{K})$  extracted from such a fit do not depend on the longitudinal velocity of the observer system in which the correlation function is measured; they are invariant under longitudinal boosts. Their physical interpretation is easiest in terms of coordinates measured in the frame where  $v(\mathbf{K})$  vanishes. There they are given by [13]

$$R_\perp^2(\mathbf{K}) = R_s^2(\mathbf{K}) = \langle \tilde{y}^2 \rangle, \quad (17a)$$

$$R_\parallel^2(\mathbf{K}) = \left\langle \left( \tilde{z} - \frac{\beta_l}{\beta_\perp} \tilde{x} \right)^2 \right\rangle - \frac{\beta_l^2}{\beta_\perp^2} \langle \tilde{y}^2 \rangle \approx \langle \tilde{z}^2 \rangle, \quad (17b)$$

$$R_0^2(\mathbf{K}) = \left\langle \left( \tilde{t} - \frac{1}{\beta_\perp} \tilde{x} \right)^2 \right\rangle - \frac{1}{\beta_\perp^2} \langle \tilde{y}^2 \rangle \approx \langle \tilde{t}^2 \rangle, \quad (17c)$$

where in the last two expressions the approximation consists of dropping terms which were found in [13] to be generically small (an extensive and quantitative discussion of this point will follow elsewhere [20]). The first expression (17a) remains true in any longitudinally boosted frame, and we will therefore now concentrate on the other three YKP parameters.

Eq. (17c) shows that the YKP parameter  $R_0(\mathbf{K})$  essentially measures the time duration  $\Delta t(\mathbf{K})$  during which particles of momentum  $\mathbf{K}$  are emitted, in the frame where the YKP velocity  $v(\mathbf{K}) = 0$ . The crucial point here is that the smallness of the difference  $\langle \tilde{x}^2 - \tilde{y}^2 \rangle$  is already accounted for directly by the fit, and no potentially small prefactor  $\beta_\perp^2$  occurs. This means that the extraction of  $\Delta t(\mathbf{K})$  from the YKP-parameter  $R_0(\mathbf{K})$  is much more direct and subject to less statistical uncertainties than in the standard fit. Clearly, this point is only true and our suggestive simple spatio-temporal interpretation of the YKP parameters is

only valid as long as the approximations in (17) are justified. For realistic emission functions they are as we shall show below.

Eqs. (17) were written down [13] in the special frame where  $v(\mathbf{K}) = 0$  which we call *Yano-Koonin (YK) frame*. In [13] it was shown that for a large class of models this frame essentially coincides with the longitudinal rest frame of the fluid cell around the point  $\bar{x}(\mathbf{K})$  of maximum emissivity at momentum  $\mathbf{K}$  (i.e. the *Longitudinal Saddle Point System* LSPS [10]). This was true also for sources which are not longitudinally boost-invariant and for which the LSPS and the LCMS (the *Longitudinally CoMoving System* in which the pion pair has  $\beta_l = 0$  [14]) do not coincide.

We now give model independent expressions, similar to Eqs. (11), (13) and (14), for the YKP fit parameters in an arbitrary observer frame. They are again given in terms of second order moments of the source function  $S(x, K)$  and thus calculable by simple quadrature. The expression for  $v(\mathbf{K})$  can then easily be used to establish, analytically and numerically, the relationship between the YK frame and the various other frames mentioned above.

We introduce the following notational shorthands:

$$A = \left\langle \left( \tilde{t} - \frac{\tilde{\xi}}{\beta_{\perp}} \right)^2 \right\rangle, \quad (18a)$$

$$B = \left\langle \left( \tilde{z} - \frac{\beta_l}{\beta_{\perp}} \tilde{\xi} \right)^2 \right\rangle, \quad (18b)$$

$$C = \left\langle \left( \tilde{t} - \frac{\tilde{\xi}}{\beta_{\perp}} \right) \left( \tilde{z} - \frac{\beta_l}{\beta_{\perp}} \tilde{\xi} \right) \right\rangle, \quad (18c)$$

where  $\tilde{\xi} \equiv \tilde{x} + i\tilde{y}$  and  $\langle \tilde{y} \rangle = \langle \tilde{x}\tilde{y} \rangle = 0$  for azimuthally symmetric sources such that  $\langle \tilde{\xi}^2 \rangle = \langle \tilde{x}^2 - \tilde{y}^2 \rangle$ . In terms of these expressions one finds

$$v = \frac{A+B}{2C} \left( 1 - \sqrt{1 - \left( \frac{2C}{A+B} \right)^2} \right), \quad (19a)$$

$$R_{\parallel}^2 = \frac{1}{2} \left( \sqrt{(A+B)^2 - 4C^2} - A + B \right) = B - vC, \quad (19b)$$

$$R_0^2 = \frac{1}{2} \left( \sqrt{(A+B)^2 - 4C^2} + A - B \right) = A - vC, \quad (19c)$$

The Yano-Koonin velocity  $v$  is zero in the frame where the expression (18c) for  $C$  vanishes

[13]; this fixes also the sign in front of the square root in (19a). For small values of  $C$  the Yano-Koonin velocity is given approximately by

$$v \approx \frac{C}{A+B} \approx \frac{\langle \tilde{z}\tilde{t} \rangle}{\langle \tilde{t}^2 \rangle + \langle \tilde{z}^2 \rangle}, \quad (20)$$

where in the second approximation we again neglected generically small terms [13] proportional to  $\langle \tilde{z}\tilde{x} \rangle$ ,  $\langle \tilde{x}\tilde{t} \rangle$ , and  $\langle \tilde{x}^2 - \tilde{y}^2 \rangle$ . The accuracy of the approximate expression (20) for  $v(\mathbf{K})$  was tested numerically and found to be excellent in the situations discussed below. In the same limit the expressions for  $R_0^2$  and  $R_{\parallel}^2$  simplify to  $R_0^2 \approx A$  and  $R_{\parallel}^2 \approx B$ , in agreement with (17).

It is instructive to compare the standard and YKP forms, Eqs. (12) and (15), for the two-particle correlation function. One finds Eq. (17a) and

$$R_{\text{diff}}^2 = R_o^2 - R_s^2 = \beta_{\perp}^2 \gamma^2 (R_0^2 + v^2 R_{\parallel}^2) \quad (21a)$$

$$R_l^2 = (1 - \beta_l^2) R_{\parallel}^2 + \gamma^2 (\beta_l - v)^2 (R_0^2 + R_{\parallel}^2) \quad (21b)$$

$$R_{ol}^2 = \beta_{\perp}^2 (-\beta_l R_{\parallel}^2 + \gamma^2 (\beta_l - v)^2 (R_0^2 + R_{\parallel}^2)) \quad (21c)$$

To invert this set of equations we calculate (cf. Eqs. (18))

$$A = \frac{1}{\beta_{\perp}^2} R_{\text{diff}}^2, \quad (22a)$$

$$B = R_l^2 - \frac{2\beta_l}{\beta_{\perp}} R_{ol}^2 + \frac{\beta_l^2}{\beta_{\perp}^2} R_{\text{diff}}^2, \quad (22b)$$

$$C = -\frac{1}{\beta_{\perp}} R_{ol}^2 + \frac{\beta_l}{\beta_{\perp}^2} R_{\text{diff}}^2. \quad (22c)$$

Inserting this into Eqs. (19) gives very cumbersome expressions which provide little physical insight. Thus, while the standard HBT radii are easily obtained from the YKP parameters via (21), the converse is not true. This indicates that the YKP parameters are more “physical” than the standard HBT radii. Nevertheless, the relations (21) provide a powerful consistency check on the experimental fitting procedure of the correlation function, of similar value as the relation [13,11]  $\lim_{K_{\perp} \rightarrow 0} (R_o(\mathbf{K}) - R_s(\mathbf{K})) = 0$  which results from azimuthal symmetry.

We now discuss numerically the dependence of the YKP parameters on the pair momentum  $\mathbf{K}$ . For our study we use the model of Ref. [13] for a finite expanding thermalized source

$$S(x, K) = \frac{M_{\perp} \cosh(\eta - Y)}{(2\pi)^3 \sqrt{2\pi(\Delta\tau)^2}} \times \exp \left[ -\frac{K \cdot u(x)}{T} - \frac{(\tau - \tau_0)^2}{2(\Delta\tau)^2} - \frac{r^2}{2R^2} - \frac{(\eta - \eta_0)^2}{2(\Delta\eta)^2} \right]. \quad (23)$$

Here  $r = \sqrt{x^2 + y^2}$ , the spacetime rapidity  $\eta = \frac{1}{2} \ln[(t + z)/(t - z)]$  and the longitudinal proper time  $\tau = \sqrt{t^2 - z^2}$  parametrize the spacetime coordinates  $x^\mu$ , with measure  $d^4x = \tau d\tau d\eta r dr d\phi$ .  $Y = \frac{1}{2} \ln[(1 + \beta_l)/(1 - \beta_l)]$  and  $M_{\perp} = \sqrt{m^2 + K_{\perp}^2}$  parametrize the longitudinal and transverse components of the pair momentum  $\mathbf{K}$ .  $T$  is the freeze-out temperature,  $R$  is the transverse geometric (Gaussian) radius of the source,  $\tau_0$  its average freeze-out proper time,  $\Delta\tau$  the mean proper time duration of particle emission, and  $\Delta\eta$  parametrizes [7] the finite longitudinal extension of the source. The expansion flow velocity  $u^\mu(x)$  is parametrized as

$$u^\mu(x) = (\cosh \eta \cosh \eta_t(r), \sinh \eta_t(r) \mathbf{e}_r, \sinh \eta \cosh \eta_t(r)), \quad (24)$$

with a boost-invariant longitudinal flow rapidity  $\eta_t = \eta$  and a linear transverse flow rapidity profile

$$\eta_t(r) = \eta_f \left( \frac{r}{R} \right). \quad (25)$$

$\eta_f$  scales the strength of the transverse flow. Other possible features of the source, like spatial and temporal gradients of the freeze-out temperature [10], other freeze-out hypersurfaces or different flow profiles, will be discussed elsewhere.

For the numerical calculations in this letter we have selected one fixed set of source parameters:  $R = 3$  fm,  $\tau_0 = 3$  fm/ $c$ ,  $\Delta\tau = 1$  fm/ $c$ ,  $\Delta\eta = 1.2$ ,  $T = 140$  MeV. We study only pion correlations and set  $m = m_{\pi} = 139$  MeV/ $c^2$ . Results for different parameter sets as well as for kaon correlation functions will be presented in a longer paper [20].

In Fig. 1 we show the relationship between the YK frame and the LCMS and LSPS.  $Y$  is the pion pair rapidity (and thus the rapidity of the LCMS),  $Y_{\text{YK}}(Y, K_{\perp})$  the rapidity of the Yano-Koonin rest frame, and  $Y_{\text{LSPS}}(Y, K_{\perp})$  the rapidity of the longitudinal rest frame of the point  $\bar{x}(Y, K_{\perp})$  of maximum emissivity (all rapidities are measured relative to the CMS of the source). For pion pairs with large  $K_{\perp}$  both the YK rest frame and the LSPS rapidities approach the LCMS rapidity  $Y$ , i.e. in this limit all the pions are emitted from a small region in the source which moves with the same longitudinal velocity as the pion pair. For small  $K_{\perp}$  the YK frame is considerably slower than the LCMS, but faster than the LSPS. *The linear relationship between the rapidity  $Y_{\text{YK}}$  of the Yano-Koonin frame and the pion pair rapidity  $Y$  is a direct reflection of the boost-invariant longitudinal expansion flow.* Such a behaviour, and thus direct evidence for a strong longitudinal expansion of the source, was recently found experimentally in Mg+Mg collisions at 4.4 A GeV/c in Dubna [21].

The difference between  $Y_{\text{YK}}$  and  $Y_{\text{LSPS}}$  is due to a longitudinal asymmetry of the source around the saddle point  $\bar{x}(Y, K_{\perp})$ ; if the source is  $z$ -symmetric around  $\bar{x}(Y, K_{\perp})$  the YK rest frame and the LSPS become identical [20]. Both  $Y_{\text{YK}}$  and  $Y_{\text{LSPS}}$  exhibit only a very weak dependence on the transverse flow of the source; its origin will be discussed quantitatively in [20].

In Fig. 2 we show  $R_0$  and  $R_{\parallel}$  as a function of  $K_{\perp}$  for pion pairs with momentum  $Y = 0$  and  $Y = 3$  in the CMS frame and compare these radii with the approximations  $R_{\parallel} \approx \sqrt{\langle \tilde{z}^2 \rangle}$ ,  $R_0 \approx \sqrt{\langle \tilde{t}^2 \rangle}$  given in Eqs. (17b,c). The approximation is seen to be *exact for vanishing transverse flow*,  $\eta_f = 0$  (as already pointed out in [13]). For  $R_{\parallel}$  it remains rather accurate for all  $K_{\perp}$ -values even in the presence of large transverse flow (Fig. 2d). The parameter  $R_0$ , on the other hand, is an accurate measure of  $\Delta t$  only for small  $K_{\perp}$  or sufficiently small transverse flow [13]. The difference between these two quantities arises from the terms  $-2\langle \tilde{x}\tilde{t} \rangle / \beta_{\perp} + \langle \tilde{x}^2 - \tilde{y}^2 \rangle / \beta_{\perp}^2$  which were neglected in the second equality of Eq. (17c). As seen in Figs. 2a,b, these terms can become a serious source of error in the determination of  $\Delta t(\mathbf{K})$  (in our case an overestimate of up to 50% in the most unfavorable case) if the transverse flow of the source is very large and not independently known such that it could be corrected for.

However, it should be noted [13] that the contamination by these undesired terms is absent for pion pairs with small pair momentum  $\mathbf{K}$ , and that therefore the determination of  $\Delta t$  from the YKP-parameter  $R_0$  is particularly clean in the region where its extraction from the standard fit according to Eq. (14) is difficult due to the  $\beta_\perp^2$ -prefactor. Furthermore, the terms that contaminate  $R_0^2$  at large  $\mathbf{K}$  and large transverse flow affect the extraction of  $\Delta t$  from Eq. (14) in exactly the same way. This problem can thus not be avoided by selecting either the standard or the YKP fitting procedure; by doing and comparing both, in particular also for heavier particles, it may be possible to estimate the amount of transverse flow and correct for it [20]. Here it should suffice to say that the associated relative error on  $\Delta t$  is everywhere less than 25% for transverse expansion velocities  $\eta_f \leq 0.3$  which we believe to be realistic, and that it should decrease for more realistic larger values for the model parameter  $\Delta\tau$  than the 1 fm/c chosen in Fig. 2.

Both the longitudinal region of homogeneity  $\sqrt{\langle \tilde{z}^2 \rangle}$  and the effective lifetime  $\sqrt{\langle \tilde{t}^2 \rangle}$  of the source decrease for pion pairs with large momenta in the CMS of the source. Asymptotically the effective lifetime becomes equal to the model parameter  $\Delta\tau = 1$  fm/c, but low-momentum pions see a much larger value. This is because for low pair momenta the longitudinal region of homogeneity  $R_\parallel$  is large, and the correlation function receives also contributions from regions freezing out at later times  $t = \sqrt{\tau_0^2 + z^2} \pm \Delta\tau$  along the surface of constant proper time  $\tau_0$  ( $z$  and  $t$  measured in the YK rest frame). This is a generic effect which should also appear for different source models [22]. The resulting strong variation of  $\langle \tilde{t}^2 \rangle$  at small  $K_\perp$  is again hard to extract from the standard fit because this region is suppressed by the factor  $\beta_\perp^2$  in Eq. (14). Although for large transverse flow  $R_0$  does no longer exactly trail the lifetime  $\langle \tilde{t}^2 \rangle$ , it clearly reflects this strong  $\mathbf{K}$ -dependence of the latter at small values of  $\mathbf{K}$ .

To summarize, we have given model-independent expressions and a detailed physical interpretation of the fit parameters for a Yano-Koonin-Podgoretskii fit to the two-particle correlation function. We have also established a simple analytical relation between these parameters and the “standard” HBT radius parameters which provides a powerful consistency

check on the experimental fitting procedure. We clarified the relationship between the YK rest frame and the previously introduced LCMS and LSPS frames and argued that the YKP fit parameters provide the most intuitive characterization of the local geometric and dynamical space-time characteristics of the source. An increase of the YK velocity with the pair rapidity signals longitudinal expansion of the source. We also pointed out a strong generic  $\mathbf{K}$ -dependence of the effective duration of particle emission which results mainly from the fast longitudinal expansion of the source, but is also modulated by transverse expansion. We hope that all these predictions will soon be checked experimentally in relativistic heavy ion collisions.

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## FIGURES

FIG. 1. (a) The rapidity of the YK frame as a function of the pion pair rapidity  $Y$  (both measured in the CMS frame of the source), for various values of the transverse momentum  $K_\perp$  of the pair and two values for the transverse flow rapidity  $\eta_f$ . (b) Same as (a), but shown as a function of  $K_\perp$  for different values of  $Y$ . The curves for negative  $Y$  are obtained by reflection along the abscissa. (c) The difference  $Y_{\text{YK}} - Y_{\text{LSPS}}$  between the rapidity of the YK frame and the longitudinal rest system of the saddle point, plotted in the same way as (a). (d) Same as (c), but shown as a function of  $K_\perp$  for different values of  $Y$ .

FIG. 2. (a)  $R_0$  and  $\sqrt{\langle \tilde{t}^2 \rangle}$  as a function of  $M_\perp$  for three values of the transverse flow rapidity  $\eta_f$ , for pion pairs with rapidity  $Y = 0$  in the source CMS frame. The lifetime  $\sqrt{\langle \tilde{t}^2 \rangle}$  is evaluated in the YK rest frame (which in this case coincides with the CMS frame). (b) Same as (a), but for pions with rapidity  $Y = 3$  in the CMS frame. (c) and (d): Same as (a) and (b), but for  $R_\parallel$  and the longitudinal length of homogeneity  $\sqrt{\langle \tilde{z}^2 \rangle}$  in the YK rest frame. For  $Y = 0$ ,  $R_\parallel$  and  $\sqrt{\langle \tilde{z}^2 \rangle}$  agree exactly because  $\beta_l = 0$  in the YK frame.



